PACKING LEVEL

Preliminary round assignment for the 27th Mathematics Alympiad

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WISKUNDE
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Colophon

The Mathematics Alympiad (Wiskunde Alympiade) is an initiative of the Freudenthal Institute, Utrecht University

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Guide for the preliminary round assignment for the 2015/2016 Mathematics Alympiad

This Mathematics Alympiad assignment consists of seven assignments and two final assignments. The first seven assignments are a run-up to the final assignments: all knowledge and insights gained in these assignments can be applied in the final assignment.

General advice for working on this assignment:

- First read the full text of the assignment so you will know what you have to do.
- Keep an eye on the time you spend on the first seven assignments; take plenty of time for the final assignments, at least two hours.
  If you divide up tasks within your team, discuss the results with each other after every assignment.

Research questions

- Several questions mention 'research…'
  In this type of questions always indicate in detail what you researched, if necessary research simpler problems, go beyond "just answering the question", research alternatives. These are criteria that your elaboration will be judged on.
- Always state precisely what you investigated, if necessary investigate simpler problems, go beyond just answering the question, look into alternatives. These are the criteria on which the quality of your result will be judged.
- If you adapt certain methods or procedures while working on the assignments, describe your adaptations in your report and include why you made them.
- It may be a good idea to use Excel or another spreadsheet program for this assignment.

Handing in:

- Final assignment A and B
- Assignments 1 to 7 as attachment(s)

The jury will receive a digital copy of your work. If you have any appendices with your work, hand in everything in a zipped folder. Include the name of your school *and* your own names in the file name.

Judging:

These are some of the points that will be considered:

- Legibility and clarity of the final assignment;
- How complete the work is;
- The use of maths;
- The argumentation used and justifying choices that have been made;
- The depth to which the various assignments have been answered;
- Presentation: form, coherence, legibility, illustrations, etcetera;
- (Mathematical) creativity in your elaboration of the assignments.

Have fun and good luck!

Introduction

You no longer have to go outside to shop: you order online and all your purchases are delivered. No more heavy bags, you just have to take the package from the postman. Sometimes the packaging is surprisingly large. You order a screwdriver, but the size of the box makes you suspect there's a complete electric drill inside. In this assignment you will research why that is done, and what the most effective way is to package orders.

Packing level

The packing level is the percentage of the content of a box used for the article.

If a small item is packed in a relatively large box, there is a lot of empty space: the box has more space filled with air (or filler) than with the item itself. In that case the packing level is low.

Is that a problem? A low packing level obviously raises the costs for packing materials (and filler), but it also raises transportation costs. If the boxes are larger than necessary, fewer will fit in a container or truck. So the packing level of the container or truck is also lower than it could be. This means that more loading, driving and unloading are necessary. And that costs more money.

Even so, mail-order firms do not opt for using a suitable box for every article. Using many different boxes means extra time for packing, because the packers need to look for the right box every time. Also, all these different boxes need to be available, and that takes storage space. Therefore the number of different boxes that a mail-order company has at hand will be limited..

The question now is: what sizes do the boxes need to be to have an optimal packing level for the various articles?

In this assignment we will work from a simplified situation: we assume that every box will contain only one article. And we assume that the optimal shipping box for an article is a beam-shaped one that fits exactly around the article. In that case the packing level is 100%.

Starting assignments

A new mail-order company has started in Amberhavn: AmberFast. To begin with, AmberFast has only three articles in its assortment:

- A coffee maker (dimensions 34 by 27 by 21 cm)
- A blender (38 by 15 by 14 cm)
- A hand mixer (20 by 15 by 9 cm)

You will look for the dimensions of one beam-shaped shipping box that will fit the coffee maker, the blender and the hand mixer on their own.

Assignment 1
Determine the dimensions of this box and determine its packing level for each of the three articles separately.

AmberFast wants a higher packing level. Therefore they decide to have two different boxes. The question is what the best dimensions are for these two different (beam-shaped) boxes, so that they will have the highest possible packing level.

Assignment 2
Research what the optimal dimensions for the two boxes are and calculate the packing level for these boxes for the individual articles on their own.

Normally, a company sells different numbers of different articles. So the packing level of a box for one specific article doesn't say that much about the efficiency of the packing process. What is needed is a new measure that combines the packing level for all articles and all boxes.

Assignment 3
Think of a way to achieve such a combined packing level (for the boxes from assignment 2) and calculate it for three situations:

I. The same amount is sold for all articles.
II. The proportion of the numbers of coffee makers: blenders : hand mixers equals 1 : 2 : 5
III. The proportion of the numbers of coffee makers: blenders: hand mixers equals 5 : 1 : 2

Assignment 4
Now research for each of the three situations above for which dimensions of the two boxes the combined packing level is optimal.

From now on, packing level refers to the combined packing level.

Not every article is ordered with the same frequency. That may influence the choice of the boxes' dimensions, as you noticed above.
Perhaps you have adapted your method to calculate the packing level along the way.

Assignment 5
Describe your final **method** to calculate the (combined) packing level. If necessary, indicate what you changed.
The description must be written so that it is understandable to everybody and usable for an arbitrary company. Also use this method in the further assignments below.

Further assignments

You have found that the proportion of ordered articles influences the optimal dimensions of the shipping boxes.

Assignment 6
Formulate a number of general rules to determine the dimensions of the two boxes for the three articles that will result in the best packing level. **Tip:** Vary the proportion of the ordered articles to check which sizes of boxes you need each time.

In addition to the coffee maker, the blender and the hand mixer, AmberFast will sell a fourth article: an espresso maker with dimensions 45 by 38 by 29 cm.

Assignment 7
The company still wants to achieve an optimal packing level with two boxes. Research whether that can be done with the 'existing' boxes, or that it makes sense to use two different boxes – and whether the proportion of ordered articles makes a difference here.

Final assignments

AmberFast continues to grow, and eventually sells over twenty different articles: coffee makers, toasters, blenders, mixers, etcetera. These appliances and their dimensions are listed in the attached Excel document.

To package these articles optimally it is important to make the right choices. Not just for the dimensions of the boxes, but also to decide how many different boxes will be used. If there is one type of box, the combined packing level will probably not be very high, while the packing level will be 100% if twenty different boxes are used, because every article has a suitable, fitting box. The rule is: the fewer types of box are needed, the better – but also: the higher the packing level, the better. So choices will have to be made.

Part A:

Assume that you may use a maximum of five boxes, and that there is the same number of orders for each article (so equal proportions).
Think of an approach in this situation to determine the optimal dimensions of the five different boxes. Keep in mind: the goal is to describe the approach, not the outcome.

Part B:

The AmberFast company wants insight in the relation between number of boxes used and the packing level, so that they can make a responsible choice. The number of orders for each article can of course vary for this. Design a method or overview in which you show how the number of different boxes relates to the combined packing level and to the proportion of the number of orders for the various articles. Describe this in a report for the AmberFast mail-order company. The company has to be able to decide for itself on the number of different boxes to be used and on their dimensions, on the basis of this report. In other words: you do not decide for the mail-order company, but you present a clear method that allows the AmberFast board to take a well-considered decision.