# “Jumping (for joy in) the queue”

**Preliminary round assignment for the29th Mathematics Alympiad – November 17, 2017**

**Colophon**

The Mathematics Alympiad (Wiskunde Alympiade) is an initiative of the Freudenthal Institute, Utrecht University. The Alympiad committee is responsible for the organisation of the Alympiad and for producing the assignment.

The committee consists of:

Eric van Dijk
Lorentz Casimir Lyceum Eindhoven

Tom Goris
Fontys Lerarenopleiding, Tilburg

Dédé de Haan
Freudenthal Institute, Utrecht & NHL Hogeschool, Leeuwarden

Senta Haas
Städtisches Gymnasium Hennef, Hennef, Duitsland

Jacques Jansen
Strabrecht College, Geldrop

Kim Kaspers
Murmellius Gymnasium, Alkmaar

Johan van de Leur,
Mathematisch Instituut, Universiteit Utrecht

Ruud Stolwijk
CITO, Arnhem & Vrijeschool Zutphen VO

Monica Wijers
Freudenthal Institute, Utrecht

Secretariat:
Liesbeth Walther en Mariozee Wintermans
Freudenthal Institute, Utrecht

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**Guide for the preliminary round assignment for the 2017/2018 Mathematics Alympiad**

This Mathematics Alympiad assignment consists of three parts: part 1 (part 1a and part 1b, with seven initial assignments), part 2 graphical representation (with four assignments) and part 3: Application (with an assignment and a final assignment). The (initial) assignments from parts 1 and 3 serve as a run up to part 3: all the knowledge and insights that have been acquired in these assignments will be needed to successfully complete the final assignment!

**General advice for working on this assignment:**

- First read the full text of the assignment so you will know what you have to do.
- Keep an eye on the time you spend on the initial assignments; take plenty of time for part 3, at least three hours. If you divide up tasks within your team, discuss the results with each other after every assignment.
- For many of the assignments, it may be a good idea to first try out a numbers or calculation example to better understand the assignment.
- If you adapt certain approaches, methods or procedures while working on the assignments, describe your adaptations in your report and include why you made them.
- It may be a good idea to use Excel or another spreadsheet program for this assignment.

***Handing in:***

- The final assignment
- The (initial) assignments 1 to 11 as well as the analysis assignments as attachments

**The jury will receive a digital copy of your work. If you have any appendices with your work, hand in everything in a zipped folder. Include the name of your school *and* your own names in the file name.**

***Judging:***

These are some of the points that will be considered:

- Legibility and clarity of the final assignment;
- How complete the work is;
- The use of maths;
- The argumentation used and justifying choices that have been made;
- The depth to which the various assignments have been answered;
- Presentation: form, coherence, legibility, illustrations and use of appendices;
- (Mathematical) creativity in your elaboration of the assignments.

**Have fun and good luck!**

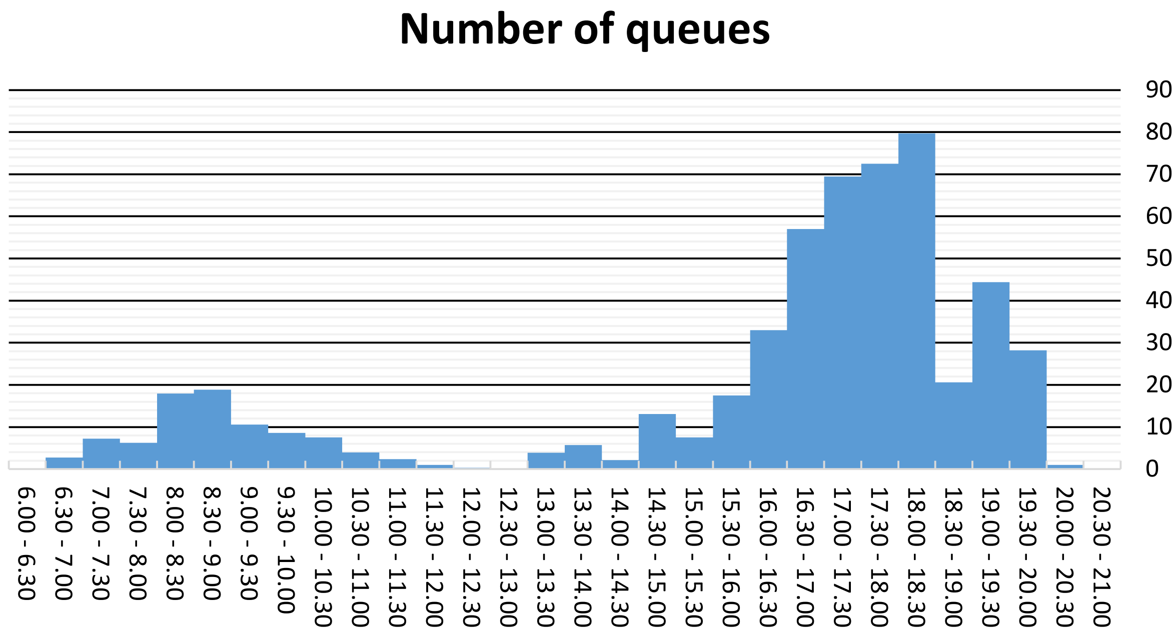
### Introduction

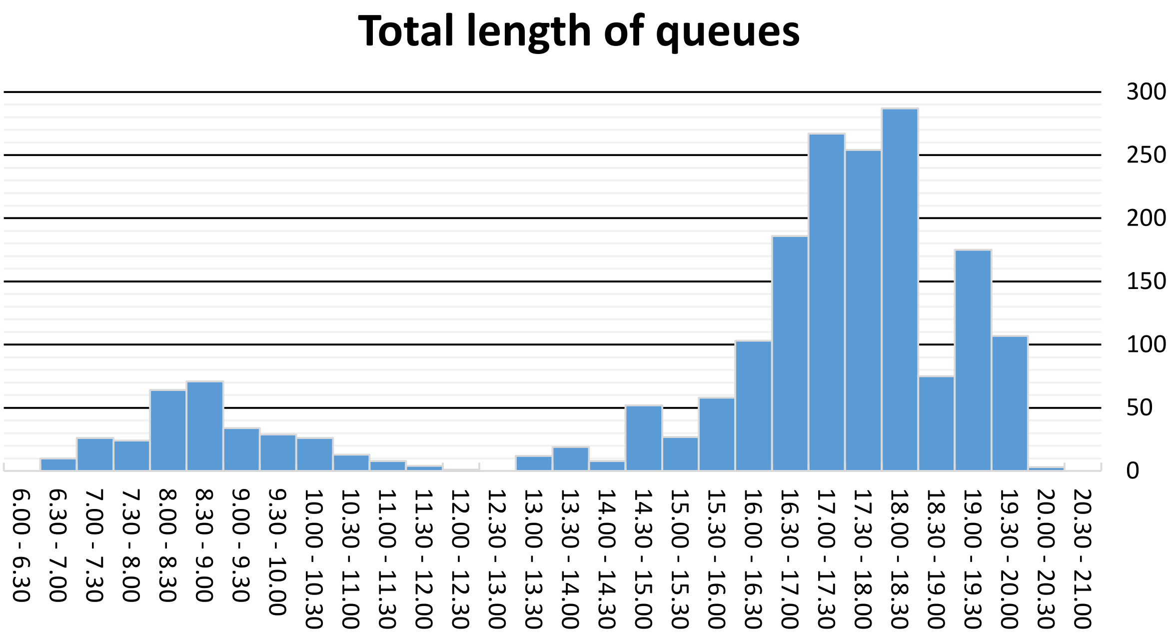
Queueing on the motorway. Traffic jams have become part of our daily lives, especially around larger towns. No one wants to be in one, but time and again most people will choose to do so after all. This leads to familiar radio traffic reports like "*At the moment there are 12 queues with a total length of 44 kilometres. There's a 2 kilometre tailback of slow-moving and stationary traffic at the Ambergroen junction."*

In this Alympiad assignment you will examine the 'queue' phenomenon, using a number of 'technical terms'. Through several initial assignments you will learn to handle these terms. Eventually you will have to use them in the final assignment.

### Part 1a: Initial – congestion weight

On the website of the Amberhavn traffic service data on queue formation are kept. Below you can see graphs for the number and total length of queues in Amberhavn on October 5 of this year.






1. What was the average length of a queue between 17.00 and 19.00 hrs in Amberhavn on October 5 of this year?

Not every queue is the same. Being in a 3 kilometre queue for 2 hours, is a different experience than being in a 6 kilometre queue for 20 minutes. The length of the queue in kilometres, multiplied by the duration in minutes is called the **congestion weight**.

The rule is: $\text{length queue (km)} \times \text{duration queue (min)} = \text{congestion weight}$

While the graphs do not allow you to read off how long a particular person spent in a queue of a particular length, you can read off that for example between 14.30 and 15.00 hrs there was a total of around 50 km of traffic jam (divided over 13 queues).

2. Using both graphs, determine the total congestion weight in Amberhavn during the evening rush hour on October 5.

**Part 1b: Initial – When is there a queue?**

We refer to **free flow** when traffic can move on the road without experiencing delays. The Amberhavn Traffic Service uses the definition that ‘queue’ is a catch-all term for three types of traffic congestion:

- **Slow moving traffic**: traffic that does not move faster than 50 km/h, but generally faster than 25 km/h, over a distance of at least 2 kilometres;
- **Stationary traffic**: traffic that almost entirely moves slower than 25 km/h over a distance of at least 2 kilometres;
- **Slow moving to stationary traffic**: slow moving traffic over a generally longer distance, with ‘groups’ of stationary traffic.

To be able to reason for yourself what factors you need to take into account for a queue to develop, we will look at an exemplary road. At night the road is mostly empty and there is no queue. Every car has enough room to drive at the maximum legal speed, without experiencing congestion. If more cars are added, this is often still the case – though there is a limit, namely when there are more cars than the road can “handle ".

Which factors do play a part in determining when a queue will form? We name some that you might take into account:

- **intensity (*I*)**: this is the number of motor vehicles per hour that passes a specific point in the road. We call the maximum value for intensity **capacity**;
- **Speed (*v*)**: this refers to the average speed of all motor vehicles on a stretch of road, in kilometres per hour;
- **density (*D*)**: this is the number of motor vehicles per kilometre on a stretch of road.

**3.** Reason that the following is the case: $\frac{I}{D} = v$

First use one or more test calculations to have some idea!

To become a bit more familiar with these concepts, we will do some calculations with them at first. We will use as an example a 20 kilometre road from A to B with one lane in both directions. This road has a capacity of 2200 cars per hour: this means that a maximum of 2200 cars per hour can drive from A to B without the traffic congesting. The following questions all involve this road from A to B.

**4.** Assume an average speed of 120 km/h. If 2000 cars per hour drive from A to B (so $I = 2000$), calculate the average amount of room each car has available.

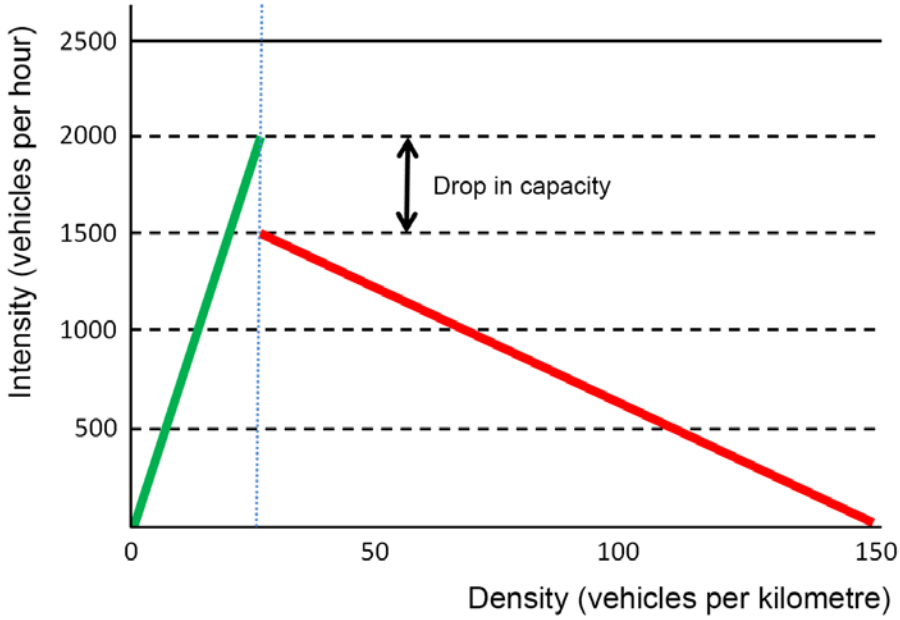
**5.** How much room is available per car, when the capacity (2200 cars per hour) is reached?

**6.** If there is a queue where all cars are at a virtual standstill, the cars can stand “bumper to bumper”. Make a realistic assumption for the length of a(n average) car and use it to calculate the average speed in a queue where cars are “bumper to bumper”.

**7.** Give an example of an intensity that is impossible on the road from A to B and of the accompanying (impossible) speed.

**Part 2: Graphical representation**

There is a difference between road capacity in “free flow” and road capacity when there is congestion. The graph below represents a linear approach of the relation between density and intensity.
The **left branch** of the graph represents the relation when there is “free flow”. The speed is constant, and density and intensity increase from left to right.
The **right branch** represents the relation when there is traffic congestion and delay, with the density increasing and the speed decreasing from left to right.




The graph shows that for many intensity values there are two different density values: one for “free flow” and one for “ congestion”. Each of these different densities will also have a (different) speed. Based on the relation between speed, density and intensity from assignment 4, you can change this graph into graphs that show the other mutual relations, for example the one between density and speed.

**8.** In the graph you can see that for a density of 25 vehicles per kilometre, the intensity drops back. Calculate the accompanying speeds, and describe as clearly as possible what the drop in capacity means in this case.

**9.** Change the given graph into one that shows the relation between density and speed.

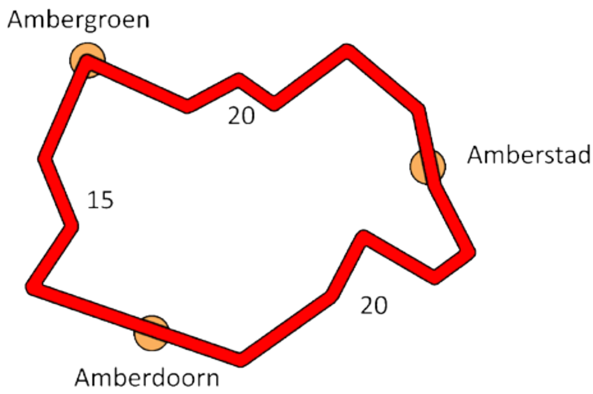
**10.** Change the given graph into one that shows the relation between speed and intensity.

**11.** Indicate as clearly as possible how to calculate the capacity of a road based on $I$, $v$ and $D$.

## Part 3 Application

Here you can find a road plan in the Amberhavn region, with the distances between the cities of Ambergroen, Amberstad and Amberdoorn in kilometres.

Below you will find the numbers of cars travelling between these cities on a week day in the morning and evening rush hour. We assume that there is one lane per direction with a maximum speed of 80 kilometres an hour.




### Morning rush hour

| From\to | Ambergroen | Amberstad | Amberdoorn |
|---|---|---|---|
| Ambergroen | | 2000 | 3000 |
| Amberstad | 1500 | | 1500 |
| Amberdoorn | 4000 | 1200 | |

### Evening rush hour

| From\to | Ambergroen | Amberstad | Amberdoorn |
|---|---|---|---|
| Ambergroen | | 5000 | 6000 |
| Amberstad | 4000 | | 1500 |
| Amberdoorn | 1000 | 1800 | |

**Assignment:** Analyse the bottlenecks in this road network.

Investigate where and when on this road network there is the largest risk of stationary traffic and delays, and how you can improve the flow of traffic. You may construct up to 30 km of extra roadway. Make use of the graphs from the initial assignments to estimate the number of cars that passes a specific point during a specific interval of time. Reason out what the intensity and density of the (current) roads will be. Also look at whether there is traffic. Use this analysis as a basis for the final assignment.

**Final assignment:** Newspaper article

Write a newspaper article focusing on the traffic problem in this road network. You will use the graphs from the part “Graphic representation”, providing an explanation so that the reader will understand what he or she can read from the graph.  You will also use at least the term congestion weight. In addition, you will also provide at least one possible solution for the congestion problems in the road network (using the option of building extra roadway). You will provide a foundation for your solution(s).